Engineering Notes

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Vortex-Lift Prediction for Complex Wing Planforms

R. G. Bradley,* C. W. Smith,† and I. C. Bhateley†
Convair Aerospace/General Dynamics, Fort Worth, Texas

THE analysis and prediction of the nonlinear lift due to the vortex flow associated with low-aspect-ratio wings has received considerable attention in the literature for many years. Methods of solution based on complex mathematical models have generally failed, and no generalized empirical methods have been developed. However, within the past several years E. C. Polhamus of the NASA Langley Research Center has proposed and verified through comparison with experimental data an analytical method for sharp-leading-edge wings of zero taper ratio.[1] The method is based on a leading-edge-suction analogy proposed by Polhamus.[2] Extension of the suction analogy to plane rectangular wings has been accomplished in work unpublished at the time of this writing by J. E. Lamar, also of NASA Langley. A method for analyzing sharp-edged, flat wings of arbitrary planform is presented here as a logical extension of the suction-analogy concept.

The well-known Polhamus method for wings with pointed tips divides the total lift into two components, a potential-flow term, $C(L_p)$, and a vortex-lift term, $C(L_v)$:

$$C_L = C_{(Lp)} + C_{(Lv)} \tag{1}$$

The full potential flow lift $C(L_p)$ is assumed to exist since the vortex induced flow reattachment maintains the Kutta condition at the trailing edge. The potential-flow term is then given by

$$C_{(Lp)} = C_{(Np)} \cos\alpha = K_p \sin\alpha \cos^2\alpha \tag{2}$$

where K_p is the normal-force slope given by potential-flow theory:

$$K_p = \frac{\partial C_{(Np)}}{\partial \sin\alpha \cos\alpha} \tag{3}$$

Computation of the vortex-lift term is based on the postulate that the vortex induced normal force on the upper surface is the same for reattached vortex flow as the leading-edge suction force for attached potential flow. The vortex-lift term for pointed tip wings is given by

$$C_{(Lv)} = C_S \cos\alpha = K_{(vLE)} \sin^2\alpha \cos\alpha \tag{4}$$

where $K(v_{LE})$ is defined by

$$K_{(vLe)} = (\partial C_S / \partial \sin^2\alpha) \tag{5}$$

The K-factors, K_p and K_v, may be computed from any accurate lifting-surface theory, e.g., vortex-lattice theory, as functions of planform and Mach number only.

In a recent unpublished work, Lamar has applied the analogy to finite tips by assuming the normal force induced by a separated tip vortex on one side to be the same as the tip-suction force on that side (side-force C_Y) for attached potential flow. The vortex-lift equation for a rectangular wing is then written as

$$C_{(Lv)} = C_T \cos\alpha + C_Y \cos\alpha = (K_{(vLE)} + K_{(vTIP)}) \sin^2\alpha \cos\alpha \tag{6}$$

where

$$K_{(vTIP)} = \frac{\partial C_Y}{\partial \sin^2\alpha} \tag{7}$$

and $K(v_{LE})$ is given by Eq. (5). As in the case for pointed-tip wings, the K-factors may be determined from an appropriate lifting-surface theory. The potential-flow force coefficients C_S, C_T, and C_Y are defined (Fig. 1) as the half-wing force coefficients nondimensionalized by the corresponding half-wing area.

Extension of the above concepts to trapezoidal planforms gives

$$C_{(Lv)} = \left(\frac{C_T}{\cos\Lambda} + C_Y - C_T \tan\Lambda\right) \cos\alpha \tag{8}$$

The above formulation, illustrated in Fig. 1c, utilizes total thrust and side force but recognizes that part of the total wing side force acts on the swept leading edge as part of the leading-edge suction vector, $C_T/\cos\Lambda$ and that remainder $(C_Y - C_T \tan\Lambda)$ acts on the wing tip.

Generalization to wings of arbitrary planform results in the following equation for the vortex-lift term:

$$C_{(Lv)} = \left(\sum_{n=1}^{N} \frac{C_{(Tn)}}{\cos\Lambda_n} + C_Y - \sum_{n=1}^{N} C_{(Tn)} \tan\Lambda_n\right) \cos\alpha \tag{9}$$

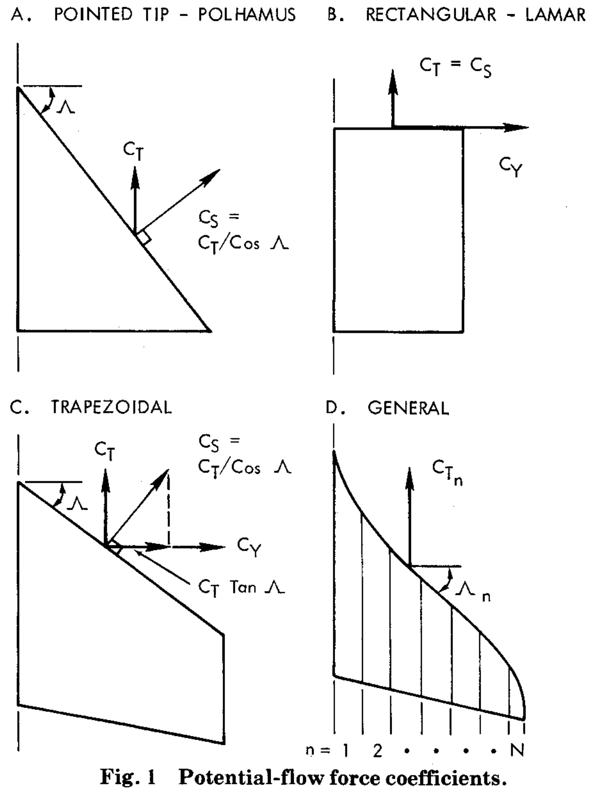

Fig. 1 Potential-flow force coefficients.

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*Design Specialist, Aerospace Technology Department. Associate Fellow AIAA.

†Senior Aerodynamics Engineer, Aerospace Technology Department.

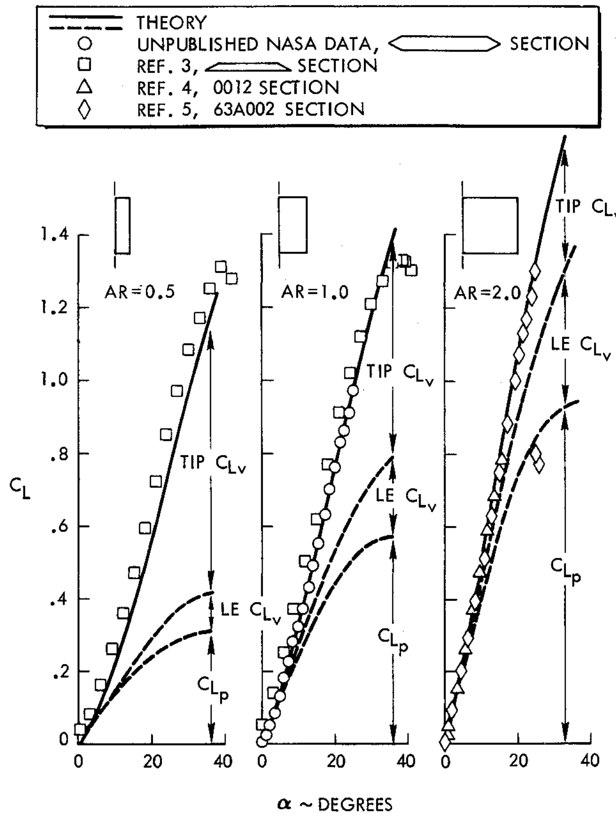

Fig. 2 Rectangular-wing lift.

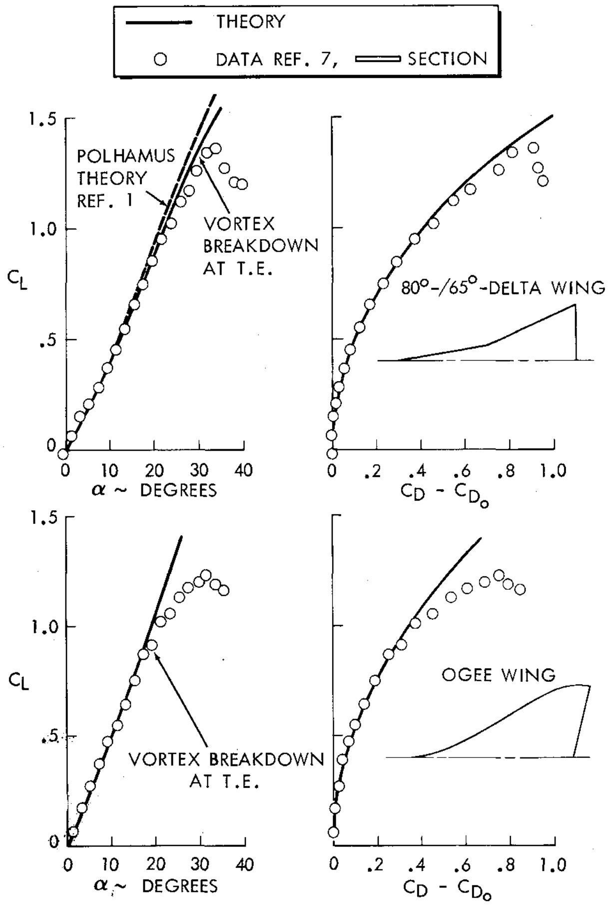

Fig. 4 Double-delta- and ogee-wing lift and drag.

The notation is illustrated in Fig. 1d. The potential-flow in-plane force coefficients, $C(T_n)$ and C_Y, are those computed from lifting-surface theory. The generalized K-factors are

$$K_{(vLE)} = \frac{\partial}{\partial \sin^2\alpha}\left(\sum_{n=1}^{N} \frac{C_{(Tn)}}{\cos\Lambda_n}\right) \tag{10}$$

$$K_{(vTIP)} = \frac{\partial}{\partial \sin^2\alpha}\left(C_Y - \sum_{n=1}^{N} C_{(Tn)} \tan\Lambda_n\right) \tag{11}$$

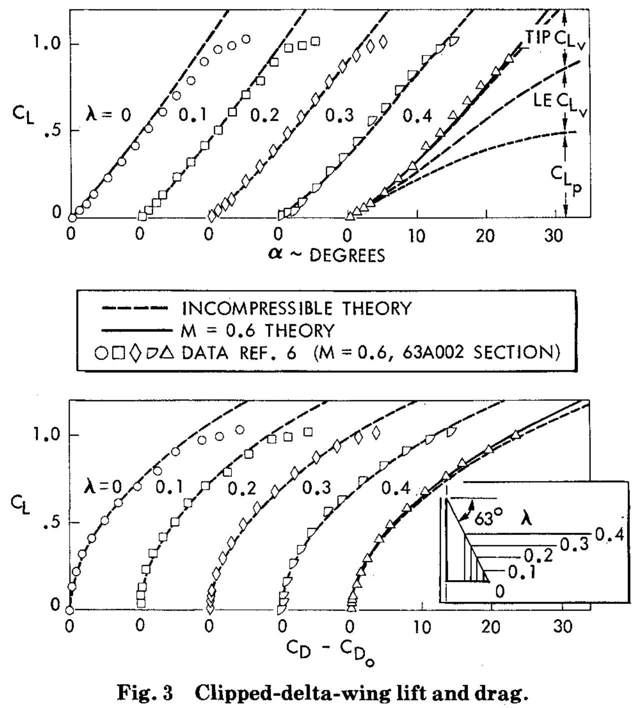

Fig. 3 Clipped-delta-wing lift and drag.

and, the total lift is given by

$$C_L = K_p \sin\alpha \cos^2\alpha + (K_{(vLE)} + K_{(vTIP)})\sin^2\alpha \cos\alpha \tag{12}$$

The extended suction-analogy theory has been applied to a variety of wing planforms. The needed potential-flow force coefficients C_N, C_T, and C_Y are computed by vortex-lattice theory.

Lift curves for three low-aspect-ratio rectangular planforms are shown in Fig. 2. A predominant tip vortex-lift increment, tip $C(L_v)$, is apparent for the low-aspect-ratio wing but becomes less significant as aspect ratio increases. Agreement with the experimental data is good at angles of attack below stall.

Lift and drag due to lift for a family of clipped-delta wings are shown in Fig. 3. The theory is noted to agree well with the lift and drag data over the range of taper ratio from 0 to 0.4. The theoretical drag due to lift is the zero-suction value, $C_L \tan\alpha$ where C_L includes the theoretical vortex lift. The lift prediction, as expected, is in exact agreement with the Polhamus theory for the delta wing, $\lambda = 0$. The lift breakdown shown for the $\lambda = 0.4$ case reveals the large tip-vortex contribution. Compressibility effects for the $\lambda = 0.4$ case were computed by application of the Goethert transformation in the vortex-lattice calculations. Improved agreement with experiment is noted in Fig. 3

Calculations for a double-delta wing and an ogee wing are compared with experimental data in Fig. 4. The agreement between theory and experiment is very good for angles of attack below that for which vortex breakdown is occurring over the wing. Points reported in Ref. 7 where breakdown reaches the wing trailing edge are shown in the figure.

Although the suction-analogy, intuitively derived by Polhamus, may lack the aesthetics of theoretical rigor, its accuracy in describing complex vortex-lift effects has been systematically verified in Ref. 1 for both subsonic and supersonic speeds. The straightforward extension of the analogy to rectangular wings by Lamar and to complex planforms as presented here lends further credence to the utility of the concept as an aerodynamic analysis tool.

References

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[3]Winter, H., "Flow Phenomena on Plates and Airfoils of Short Span," TM 798, July 1936, NACA.

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A Modified Wall Wake Velocity Profile for Turbulent Compressible Boundary Layers

Chen-Chih Sun* and Morris E. Childs†
University of Washington, Seattle, Wash.

Symbols

- A = $\{[(\gamma-1)/2]M_e^2/(T_w/T_e)\}^{1/2}$
- a = a constant, see Eq. (8)
- B = $\{(1+[(\gamma-1)/2]M_e^2)/(T_w/T_e)\}-1$
- C = constant in Law of the Wall (usually equals 5.1)
- C_f = skin friction coefficient $\tau_w/(1/2)\,\rho_e u_e^2$
- K = constant in mixing length (usually equals 0.4)
- M = Mach number
- Re_δ = Reynolds number based on δ
- u = velocity in streamwise direction
- u^* = $(u_e/A)\arcsin\{[(2A^2\,u/u_e)-B]/(B^2+4A^2)^{1/2}\}$
- u_τ = Friction velocity $(\tau_w/\rho_w)^{1/2}$
- W = Coles' universal wake function
- y = coordinate normal to wall
- γ = ratio of specific heats
- δ = boundary-layer thickness
- η = y/δ
- ν = kinematic viscosity
- Π = coefficient of wake function
- ρ = density
- σ = $[(\gamma-1)/2]M_e^2/\{1+[(\gamma-1)/2]M_e^2\}$
- τ = shear stress

Subscripts

- e = conditions at the edge of the boundary layer
- w = conditions at the wall

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*Research Associate (Postdoctoral), Department of Mechanical Engineering.

†Professor, Department of Mechanical Engineering.

A SIMPLE representation of the mean velocity distribution in a turbulent boundary layer is very useful in integral analyses of turbulent flow problems. After an extensive survey of mean velocity profile measurements, Coles[1] suggested that for incompressible turbulent boundary layer flow the velocity profile may be represented by a linear combination of two universal functions in the form

$$u/u_\tau = (1/K)\ln(yu_\tau/\delta) + C + \Pi W(y/\delta)/K = f(y) + g(y) \tag{1}$$

where

$$f(y) = (1/K)\ln(yu_\tau/\delta) + C \tag{2}$$

is the Law of the Wall and

$$g(y) = \Pi(y/\delta)/K \tag{3}$$

is the Law of the Wake.

Setting u/u_e and $W(y/\delta) = 2$ at $(y/\delta) = 1$ in Eq. (1), and subtracting the resulting equation from Eq. (1) leads to an expression for the velocity of the form

$$u/u_e = 1 + (1/K)(u_\tau/u_e)\ln(y/\delta) - (\Pi/K)(u_\tau/u_e)[(2 - W(y/\delta)] \tag{4}$$

Mathews et al.[2] have developed a wall-wake representation of the velocity profile in a form applicable for isoenergetic compressible boundary layers. Their profile is

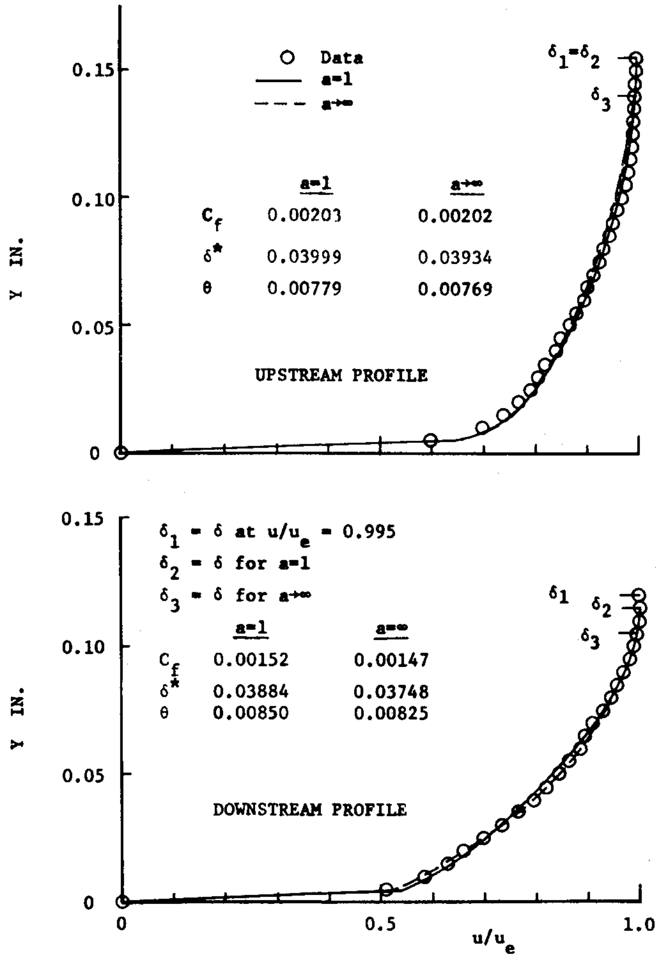

Fig. 1 Velocity profiles upstream and downstream of a shock wave-boundary layer interaction.